

Name: Section A01

1. (10 points) A jar contains a 4-sided die (a tetrahedron) and a 6-sided die. We uniformly at random pick a die out of the jar and roll it. We observe the number on the lower face of the die.

- (a) Describe the sample space.

$$\Omega = \{1, 2, 3, 4, 5, 6\}$$

- (b) What is the probability that the lower face is bigger than 3?

A: Lower face > 3

B_4 : Picked 4-sided die B_6 : Picked 6-sided die

$$\begin{aligned} P(A) &= P(A | B_4)P(B_4) + P(A | B_6)P(B_6) \\ &= \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = \boxed{\frac{3}{8}} \end{aligned}$$

- (c) Assuming the roll is a 3, what is the chance we picked the 6-sided die?

A: Lower face = 3

$$\begin{aligned} P(B_6 | A) &= \frac{P(B_6) \cdot P(A | B_6)}{P(A | B_4)P(B_4) + P(A | B_6)P(B_6)} = \frac{\frac{1}{2} \cdot \frac{1}{6}}{\frac{1}{4} \cdot \frac{1}{2} + \frac{1}{6} \cdot \frac{1}{2}} = \frac{\frac{1}{2}}{\frac{1}{4} + \frac{1}{6}} = \frac{\frac{1}{2}}{\frac{10}{24}} \\ &= \frac{24}{10} \cdot \frac{1}{6} = \boxed{\frac{2}{5}} \end{aligned}$$