

Name: Section A02

1. (10 points) A standard 52-card deck is shuffled, and two cards are dealt face down. The dealer looks at both cards.

- (a) Describe the sample space.

Let D be the set of 52 cards.

$$\Omega = \{(c_1, c_2) : c_1 \neq c_2, c_1, c_2 \in D\}$$

- (b) If the dealer tells you that at least one of the cards is an Ace, what is the probability there are two Aces?

A : Event of getting two Aces

B : Event of getting at least one Ace

$$\begin{aligned} P(A|B) &= \frac{P(A \cap B)}{P(B)} = \frac{P(A)}{P(B)} = \frac{4 \cdot 3 / 52 \cdot 51}{(4 \cdot 51 + 48 \cdot 4) / 52 \cdot 51} \\ &= \frac{4 \cdot 3}{4(51 + 48)} = \frac{3}{99} = \boxed{\frac{1}{33}} \end{aligned}$$

- (c) If the dealer tells you that one of the cards is the Ace of Spades, what is the probability there are two Aces?

C : Event that one of the cards is an Ace of Spades.

$$\begin{aligned} P(A|C) &= \frac{P(A \cap C)}{P(C)} = \frac{(3 \cdot 2) / 52 \cdot 51}{51 \cdot 2 / 52 \cdot 51} \\ &= \frac{3 \cdot 2}{51 \cdot 2} = \frac{3}{51} = \boxed{\frac{1}{17}} \end{aligned}$$