Introduction to Abstract Mathematics
MAT 108 Lecture Notes

Textbook: A Transition to Advanced Mathematics, 8th Ed.
by D. Smith, M. Eggen and R. St. Andre

Instructor: Daniel Blanquicett
Course Site: `quicetor.impa.br/Teaching`
E-mail: drbt(at)math.ucdavis.edu

Canvas: Only for Zoom, Videos and Grading
Zoom Meeting: https://ucdavis.zoom.us/j/99215204274

Summer I, 2020

Contents

Propositions and Connectives

Definition

A *proposition* is a sentence that has exactly one truth value: either true (T), or false (F).

Definition

The *negation* of a proposition P, denoted $\sim P$, is the proposition "not P". $\sim P$ is true exactly when P is false, and viceversa.

The Conjunction

Definition

Given propositions P and Q, the *conjunction* of P and Q, denoted $P \wedge Q$, is the proposition "P and Q". $P \wedge Q$ is true exactly when *both* P and Q are true.

P	Q	$P \wedge Q$
T	T	T
F	T	F
T	F	F
F	F	F

The Disjunction

Definition

Given propositions P and Q, The *disjunction* of P and Q, denoted $P \vee Q$, is the proposition "P or Q". $P \vee Q$ is true exactly when *at least one* of P or Q is true.

P	Q	$P \vee Q$
T	T	T
F	T	T
T	F	T
F	F	F

Tautology and Contradiction

Definition

A *tautology* is a propositional form that is true for every assignment of truth values to its components.
A *contradiction* is a propositional form that is false for every assignment of truth values to its components.

Example

Prove that $(P \vee Q) \vee (\sim P \wedge \sim Q)$ is a tautology.

Equivalence

Basic Laws

Theorem (1.1.1)

For propositions P, Q, and R, the following are equivalent:

(a)	P	and	$\sim(\sim P)$
(b)	$P \vee Q$	and	$Q \vee P$
(c)	$P \wedge Q$	and	$Q \wedge P$
(d)	$P \vee (Q \vee R)$	and	$(P \vee Q) \vee R$
(e)	$P \wedge (Q \wedge R)$	and	$(P \wedge Q) \wedge R$
(f)	$P \wedge (Q \vee R)$	and	$(P \wedge Q) \vee (P \wedge R)$
(g)	$P \vee (Q \wedge R)$	and	$(P \vee Q) \wedge (P \vee R)$
(h)	$\sim(P \wedge Q)$	and	$\sim P \vee \sim Q$
(i)	$\sim(P \vee Q)$	and	$\sim P \wedge \sim Q$

Examples

Denial

Conditionals

Definition

For propositions P and Q, the *conditional sentence* $P \Rightarrow Q$ is the proposition "If P, then Q". P is called the *antecedent* and Q is the *consequent*. The sentence $P \Rightarrow Q$ is true if and only if P is false or Q is true.

P	Q	$P \Rightarrow Q$
T	T	T
F	T	T
T	F	F
F	F	T

Examples

UCDAVIS
AGGIES

Converse and Contrapositive

Let P and Q be propositions. The *converse* of $P \Rightarrow Q$ is $Q \Rightarrow P$. The *contrapositive* of $P \Rightarrow Q$ is $(\sim Q) \Rightarrow (\sim P)$.

Theorem (1.2.1)

$P \Rightarrow Q$ is equivalent to its contrapositive. On the other hand, it is NOT equivalent to its converse.

The Biconditional

Definition

For propositions P and Q, the *biconditional sentence* $P \iff Q$ is the proposition "P if and only if Q". The sentence $P \iff Q$ is true exactly when P and Q have the same truth values.

P	Q	$P \iff Q$
T	T	T
F	T	F
T	F	F
F	F	T

Examples

Basic Laws

Theorem (1.2.2)

For propositions P, Q, and R, are equivalent:

(a)	$P \Rightarrow Q$	and	$\sim P \vee Q$
(b)	$P \iff Q$	and	$(P \Rightarrow Q) \wedge (Q \Rightarrow P)$
(c)	$\sim (P \Rightarrow Q)$	and	$P \wedge \sim Q$
(d)	$\sim (P \wedge Q)$	and	$P \Rightarrow \sim Q$
(e)	$\sim (P \wedge Q)$	and	$Q \Rightarrow \sim P$
(f)	$P \Rightarrow (Q \Rightarrow R)$	and	$(P \wedge Q) \Rightarrow R$
(g)	$P \Rightarrow (Q \wedge R)$	and	$(P \Rightarrow Q) \wedge (P \Rightarrow R)$
(h)	$(P \vee Q) \Rightarrow R$	and	$(P \Rightarrow R) \wedge (Q \Rightarrow R)$

Examples