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Theorem (2.1.2)

If A and B are sets with no elements, then $A = B$.

Proof: See Exercise 12. $\Diamond$

Theorem (2.1.3)

For any sets A and B, if $A \subseteq B$ and $A \neq \varnothing$, then $B \neq \varnothing$.

Power set

Definition

Let A be a set. The *power set* of A is the set whose elements are the subsets of A and is denoted $\mathscr{P}(A)$. Thus

$$\mathscr{P}(A) = \{B : B \subseteq A\}.$$

Theorem (2.1.4)

If A is a set with n elements, then $\mathscr{P}(A)$ is a set with 2^n elements.

Theorem (2.1.5)

Let A and B be sets. Then $A \subseteq B$ iff $\mathscr{P}(A) \subseteq \mathscr{P}(B)$.

Set operations

Definition

Let A and B be sets.
The *union of* A *and* B is the set $A \cup B = \{x : x \in A \text{ or } x \in B\}$.
The *intersection of* A *and* B is the set
$A \cap B = \{x : x \in A \text{ and } x \in B\}$.
The *difference of* A *and* B is the set
$A - B = \{x : x \in A \text{ and } x \notin B\}$.

Venn diagrams:

Example

For $A = \{1, 2, 4, 5, 7\}$ and $B = \{1, 3, 5, 9\}$,
$A \cup B = \{1, 2, 3, 4, 5, 7, 9\}$,
$A \cap B = \{1, 5\}$,
$A - B = \{2, 4, 7\}$,
$B - A = \{3, 9\}$.

Disjoint sets

Definition

Sets A and B are *disjoint* iff $A \cap B = \varnothing$.

Example

The sets $\{1, 2, b\}$ and $\{-1, t, n, 8\}$ are disjoint. The set of even natural numbers and the set of odd natural numbers are disjoint. The intervals $(0, 1)$ and $[1, 2]$ are disjoint, but $(0, 1]$ and $[1, 2]$ are not disjoint because they both contain the element 1.

Basic properties

Theorem (2.2.1)

For all sets A, B, and C,

a) $A \subseteq A \cup B$.
b) $A \cap B \subseteq A$.
c) $A \cap \varnothing = \varnothing$.
d) $A \cup \varnothing = A$.
e) $A \cap A = A$.
f) $A \cup A = A$.
g) $A - \varnothing = A$.
h) $\varnothing - A = \varnothing$.
i) $A \cup B = B \cup A$.
j) $A \cap B = B \cap A$.

k) $A \cup (B \cup C) = (A \cup B) \cup C$.

l) $A \cap (B \cap C) = (A \cap B) \cap C$.

m) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

n) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

o) $A \subseteq B$ iff $A \cup B = B$.

p) $A \subseteq B$ iff $A \cap B = A$.

q) If $A \subseteq B$, then $A \cup C \subseteq B \cup C$.

r) If $A \subseteq B$, then $A \cap C \subseteq B \cap C$.

Example

Is it T or F that $(A \cup B) \cap C = A \cup (B \cap C)$, for all sets A, B, C?

Complements

Definition

Let U be the universe and $A \subseteq U$. The *complement* of A is the set $A^c = U - A$.

Example

For set $A = \{2, 4, 6, 8\}$, we have $A^c = \{10, 12, 14, 16, \dots\}$ if the universe is all even natural numbers, while $A^c = \{1, 3, 5, 7, 9, 10, 11, 12, 13, \dots\}$ if the universe is $U = \mathbb{N}$.

For the universe $U = \mathbb{R}$, if $B = (0, \infty)$, then $B^c = (-\infty, 0]$. And, if $D = \{5\}$ then $D^c = (-\infty, 5) \cup (5, \infty)$.

Theorem (2.2.2)

Let U be the universe, and let $A, B \subset U$. Then

a) $(A^c)^c = A$.

b) $A \cup A^c = U$.

c) $A \cap A^c = \varnothing$.

d) $A - B = A \cap B^c$.

e) $A \subseteq B$ iff $B^c \subseteq A^c$.

f) $A \cap B = \varnothing$ iff $A \subseteq B^c$.

g) $(A \cup B)^c = A^c \cap B^c$.

h) $(A \cap B)^c = A^c \cup B^c$.