Partitions

Definition

Let A be a nonempty set. $\mathscr{P}$ is a *partition of A* iff $\mathscr{P}$ is a set of subsets of A such that

i) If $X \in \mathscr{P}$, then $X \neq \varnothing$.

ii) If $X \in \mathscr{P}$ and $Y \in \mathscr{P}$, then $X = Y$ or $X \cap Y = \varnothing$.

iii) $\bigcup_{X \in \mathscr{P}} X = A$.

Example

Prove that the family $\mathscr{G} = \{[n, n+1) : n \in \mathbb{Z}\}$ is a partition of $\mathbb{R}$.

Theorem (3.3.1)

If R is an equivalence relation on $A \neq \varnothing$, then A/R is a partition of A.

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Theorem (3.3.2)

Let $\mathscr{P}$ be a partition of $A \neq \varnothing$. Given $x, y \in A$, define xQy iff $\exists C \in \mathscr{P}$ s.t. $x \in C$ and $y \in C$. Then

1. *Q is an equivalence relation on A.*
2. *$A/Q = \mathscr{P}$.*

Functions

Functions as relations

Definition

Let A, B be sets. A *function* (or *mapping*) *from A to B* is a relation f from A to B s.t.

1. $\mathrm{Dom}(f) = A$, and
2. $[(x, y) \in f \text{ and } (x, z) \in f] \implies y = z$.

We write

$$f : A \to B,$$

and read 'f is a function from A to B', or 'f maps A to B'.
B is called the *codomain of f*.
When $B = A$, we say that f is a *function on A*.
When $(x, y) \in f$ we write

$$y = f(x).$$

We say that y is the *image of f at x* (or *value of f at x*), and that x is a *pre-image of y*.

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Theorem (4.1.1)

Two functions f and g are equal iff

1. $Dom(f) = Dom(g)$, *and*
2. $\forall x \in Dom(f),\ f(x) = g(x)$.

Definition

A function x with domain $\mathbb{N}$ is called an *infinite sequence*, or simply a *sequence*. The image of $n \in \mathbb{N}$ is usually written x_n instead of $x(n)$ and is called the nth *term of the sequence*.

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