Functions

Theorem (4.2.3)

Let $f : A \to B$. Then $f \circ I_A = f$ and $I_B \circ f = f$.

Theorem (4.2.4)

Let $f : A \to B$ *with* $Rng(f) = C$*. If* f^{-1} *is a function, then* $f^{-1} \circ f = I_A$ *and* $f \circ f^{-1} = I_C$*.*

Restrictions

Definition

Let $f : A \to B$ and $D \subseteq A$. The *restriction of f to D* is the function

$$f|_D = \{(x, y) : y = f(x) \text{ and } x \in D\}.$$

When g is a restriction of f, we say that f is an *extension* of g.

Union of functions

Theorem (4.2.5)

Let $h : A \to C$ and $g : B \to C$. If $A \cap B = \varnothing$, then $h \cup g$ is a function with domain $A \cup B$. Moreover, $f = h \cup g$ satisfies

$$f(x) = \begin{cases} h(x) & \text{if } x \in A, \\ g(x) & \text{if } x \in B. \end{cases}$$

Definition

Let $I \subseteq \mathbb{R}$ be an interval and let $f : I \to \mathbb{R}$. We say that

1. f is *increasing on I* iff $(\forall x, y \in I)[x < y \Rightarrow f(x) < f(y)]$.
2. f is *increasing on I* iff $(\forall x, y \in I)[x < y \Rightarrow f(x) > f(y)]$.

Functions that are Onto

Definition

A function $f : A \to B$ is *onto* B (or a *surjection*) if $\text{Rng}(f) = B$.

When f is a surjection, we write $f : A \xrightarrow{\text{onto}} B$.

Theorem (4.3.1)

If $f : A \xrightarrow{onto} B$ and $g : B \xrightarrow{onto} C$, then $g \circ f : A \xrightarrow{onto} C$.

Theorem (4.3.2)

Let $f : A \to B$ and $g : B \to C$. If $g \circ f$ is onto C, then g is onto C.

1-1 Functions

Definition

A function $f : A \to B$ is *one-to-one* (or is *an injection*) if $(\forall x, y \in A)[f(x) = f(y) \implies x = y]$. We write $f : A \xrightarrow{1\text{-}1} B$.

Theorem (4.3.3)

If $f : A \xrightarrow{\text{1-1}} B$ and $g : B \xrightarrow{\text{1-1}} C$, then $g \circ f : A \xrightarrow{\text{1-1}} C$.

Theorem (4.3.4)

Let $f : A \to B$ and $g : B \to C$. If $g \circ f$ is 1-1, then f is 1-1.

Theorem (4.3.5)

If $f : A \xrightarrow{\text{1-1}} B$, then every restriction of f is 1-1.

Theorem (4.3.6)

Let $h : A \to C$ and $g : B \to D$ be functions, with $A \cap B = \varnothing$.

1. *If h is onto C and g is onto D, then $h \cup g : A \cup B \xrightarrow{\text{onto}} C \cup D$.*
2. *If h is 1-1, g is 1-1 and $C \cap D = \varnothing$, then $h \cup g : A \cup B \xrightarrow{\text{1-1}} C \cup D$.*

Definition

A function $f : A \to B$ is a *one-to-one correspondence* (or a *bijection*) if f is 1-1 and onto B.