Functions

Bijections

Recall that a function $f : A \to B$ is a *one-to-one correspondence* (or a *bijection*) if f is 1-1 and onto B.

Example

Theorem (4.4.1)

If $f : A \to B$ is a bijection and $g : B \to C$ is a bijection, then $g \circ f : A \to C$ is a bijection.

Inverse Functions

Theorem (4.4.2)

Let $f : A \to B$ be a function.

a) $f^{-1} : Rng(f) \to A$ *is a function if and only if* f *is 1-1.*

b) *If* f^{-1} *is a function, then* f^{-1} *is 1-1.*

Corollary (4.4.3)

The inverse of a bijection is a bijection.

Theorem (4.4.4)

If $f : A \to B$ and $g : B \to A$ are functions, then

1. $g = f^{-1}$ *if and only if* $g \circ f = I_A$ *and* $f \circ g = I_B$.
2. *If f is a bijection, then* $g = f^{-1}$ *if and only if* $g \circ f = I_A$ *or* $f \circ g = I_B$.

Definition

Let $A \neq \varnothing$ be a set. A *permutation of A* is a one-to-one correspondence from A onto A.

Theorem (4.4.5)

Let $A \neq \varnothing$. Then

a) *the identity mapping I_A is a permutation of A.*
b) *the composite of permutations of A is a permutation of A.*
c) *the inverse of a permutation of A is a permutation of A.*
d) *if f is a permutation of A, then $f \circ I_A = I_A \circ f = f$.*
e) *if f is a permutation of A, then $f \circ f^{-1} = f^{-1} \circ f = I_A$.*
f) *if f and g are permutations of A, then $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.*

Set Images

Definition

Let $f : A \to B$ and let $X \subseteq A$ and $Y \subseteq B$. The *image of X* or *image set of X* is

$$f(X) = \{y \in B : y = f(x) \text{ for some } x \in X\}$$

and the *inverse image of Y* is

$$f^{-1}(Y) = \{x \in A : f(x) \in Y\}.$$

Theorem (4.5.1)

Let $f : A \to B$, with $C, D \subseteq A$ and $E, F \subseteq B$. Then

a) $f(C \cap D) \subseteq f(C) \cap f(D)$.

b) $f(C \cup D) = f(C) \cup f(D)$.

c) $f^{-1}(E \cap F) = f^{-1}(E) \cap f^{-1}(F)$.

d) $f^{-1}(E \cup F) = f^{-1}(E) \cup f^{-1}(F)$.