Chapter 5 July 22, 2020

Cardinality

Countable sets

Recall that S is *countable* if S is finite or denumerable.
S is *uncountable* if S is not countable.

Theorem (5.2.4)

The open interval $(0, 1)$ *is uncountable.*

Definition

A set S has *cardinality* $\mathfrak{c}$ (or *cardinal number* $\mathfrak{c}$) if S is equivalent to $(0,1)$. We write $\bar{\bar{S}} = \mathfrak{c}$, which stands for *continuum*.

Theorem (5.2.5)

a) *Every open interval* (a, b) *is uncountable and has cardinality* $\mathfrak{c}$.

b) *The set* $\mathbb{R}$ *is uncountable and has cardinality* $\mathfrak{c}$.

COUNTABLE SETS

Theorem (5.3.1)

The set $\mathbb{Q}^+$ of positive rational numbers is denumerable.

Theorem (5.3.2)

Every subset of a countable set is countable.

Corollary (5.3.3)

A set A is countable iff A is equivalent to some subset of $\mathbb{N}$.

Theorem (5.3.4)

If A is denumerable, then $A \cup \{x\}$ is denumerable.

UCDAVIS
AGGIES

Theorem (5.3.5)

If A is denumerable and B is finite, then $A \cup B$ is denumerable.

Theorem (5.3.6)

If A and B are disjoint denumerable sets, then $A \cup B$ is denumerable.

Theorem (5.3.7)

The set $\mathbb{Q}$ of all rational numbers is denumerable.

Theorem (5.3.8)

Let $\mathscr{A}$ be a countable collection of countable sets. Then $\bigcup_{A \in \mathscr{A}}$ is countable.

Corollary (5.3.9)

a) *If $\mathscr{A}$ is a finite pairwise disjoint family of denumerable sets, then $\bigcup_{A \in \mathscr{A}} A$ is countable.*
b) *If A and B are countable sets, then $A \cup B$ is countable.*
c) *If $\mathscr{A}$ is a finite collection of countable sets, then $\bigcup_{A \in \mathscr{A}} A$ is countable.*
d) *If $\mathscr{A}$ is a denumerable family of countable sets, then $\bigcup_{A \in \mathscr{A}} A$ is countable.*

UCDAVIS
AGGIES