Math 108, SS1 2020
July 10, 2020

REVIEW HOMEWORK

NAME(use CAPITAL letters, *first name first*):_____

NAME(sign):_____

ID#:_____

Instructions: Each of the 4 problems has equal worth. Read each question carefully and answer it in the space provided, or answer it in a different paper and sign that paper; it is optional to print the homework. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor when determining credit. Unless directed to do so, do *not* prove any theorem or proposition seen in class, and do not evaluate complicated expressions to give the result as a fraction or a decimal number. However, if you are using any of the problems in the textbook, then you have to solve or prove it.

To deliver: Submit your solutions on Canvas in either pdf or jpg format. Sign and submit this first page (or the paper that you are signing). Submit your self-video on Canvas as well, everything before 5:00PM.

Make sure that you have a total of 5 pages (including this one) with 4 problems.

1	
2	
3	
4	
TOTAL	

Remark: In this homework, $\mathbb{N} = \{1, 2, 3, \dots\}$, so that $0 \notin \mathbb{N}$.

1. Determine whether the following statements are True or False. Justify your answer with a proof or a counterexample as appropriate.

   (a) $[(P \iff Q) \vee \sim R] \iff [(\sim P \vee Q) \wedge (Q \implies P) \wedge (R \vee Q)]$ is a tautology.

   (b) Every $n \in \mathbb{N}$ such that $n \geq 12$ can be written in the form $2k + 5l$, for some $k, l \in \mathbb{N}$.

2. (a) Let A, B, C and D be sets such that $C \subseteq A,\ D \subseteq B$, and A and B are disjoint. Prove that C and D are disjoint.

(b) Find the power set of the set $A = \{\varnothing, \{\varnothing\}, 1, \{1, 2\}\}$.

3. Consider the relations $R = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x = y^2 + 2\}$ and $S = \{(x, y) \in \mathbb{R} \times \mathbb{R} : x = 2y\}$.

   (a) Find the relation R^{-1}.

   (b) Find the relation $S \circ R$.

   (c) Find $\text{Rng}(R^{-1} \circ S^{-1})$.

4. Consider the set $A = \mathbb{N} \times \mathbb{N}$ and the relation $R = \{((x, y), (z, w)) \in A \times A : x + y = z + w\}$.

   (a) Prove that R is an equivalence relation.

   (b) Describe A/R, and ilustrate the classes $\overline{(1,1)}, \overline{(1,2)}, \overline{(2,2)}$ and $\overline{(1,7)}$ with a picture.