Some Hints to HW#5

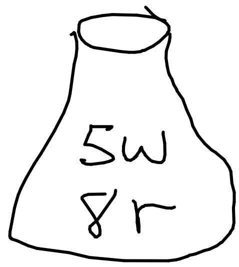
$\boxed{1}$ $X_1 = \mathbb{1}\{1^{st} \text{ ball is white}\}$ $\qquad$ $X_2 = \mathbb{1}\{2^{nd} \text{ ball is white}\}$

$$= \begin{cases} 1, & \text{if } 1^{st} \text{ ball } w \\ 0, & \text{if } 1^{st} \text{ ball } r \end{cases} \qquad = \begin{cases} 1, & 2^{nd} \text{ ball } w \\ 0, & 2^{nd} \text{ ball } r \end{cases}$$

Chosing without replacement implies $|\Omega| = 13 \times 12$

We need to fill in 2×2 table

$X_1 \backslash X_2$	0	1	f_{X_1}
0	$f(0,0)$	$f(0,1)$	?
1	$f(1,0)$	$f(1,1)$	?
f_{X_2}	?	?	1



For instance:

$$f(0,1) = \mathbb{P}(X_1 = 0, X_2 = 1) = \mathbb{P}(1^{st} \text{ ball } r,\ 2^{nd} \text{ ball } w)$$
$$= 8 \times 5 / |\Omega| \quad = \quad 10/39$$

Then compute $\mathbb{E}(X_1) = 0 \cdot f_{X_1}(0) + 1 f_{X_1}(1) = f_{X_1}(1) = ?$

Also $\mathbb{E}(X_2) = \cdots = \mathbb{E}(X_1)$, and $\mathbb{E}(X_1 X_2) = \sum_{x_1} \sum_{x_2} x_1 x_2 f_{X_1,X_2}(x_1,x_2)$
$= \cdots = f(1,1) = ?$

and $\mathrm{Var}(X_1) = \mathrm{Var}(X_2) = ?$

Finally $\rho(X_1, X_2) = \dfrac{\mathrm{Cov}(X_1, X_2)}{\mathrm{Var}(X_1)} = \cdots = -1/12$.

Does it make sense that $\rho(X_1, X_2) < 0$?

$\boxed{4}$ Since $X_2 \in \{0,1\}$, it is enough to compute $\mathbb{E}(X_1 | X_2 = 0)$ and $\mathbb{E}(X_1 | X_2 = 1)$. So, we need $f_{X_1|X_2}(x_1 | X_2 = 0)$ and $f_{X_1|X_2}(x_1 | X_2 = 1)$ for $x_1 \in \{0,1\}$.

For instance, $f_{X_1|X_2}(0|1) = \dfrac{f_{X_1,X_2}(0,1)}{f_{X_2}(1)} = \dfrac{10/39}{f_{X_2}(1)} = ?$

and so on...