Math 135A, Fall 2019
October 30, 2019

# MIDTERM EXAM 1

NAME(use CAPITAL letters, *first name first*):________________________________________

NAME(sign):________________________________

ID#:_______________________

**Instructions**: Each of the 4 problems has equal worth. Read each question carefully and answer it in the space provided. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor when determining credit. Calculators, books or notes are not allowed. You can use the Formula Sheet provided at our course website. Unless directed to do so, do *not* evaluate complicated expressions to give the result as a fraction or a decimal number. The proctor has been directed not to answer any interpretation questions.

Make sure that you have a total of 5 pages (including this one) with 4 problems.

| | |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| TOTAL | |

1. In a state lottery, a player must choose 8 of the numbers from 1 to 40. The lottery commission then performs an experiment that randomly selects 8 of these 40 numbers. What is the probability that a player has

   (a) (8pts) all 8 of the numbers selected?

   (b) (8pts) none of the 8 numbers selected?

   (c) (9pts) at least 6 of the numbers selected?

2. Urn $U_1$ contains 3 red and 3 black balls, whereas urn $U_2$ contains 4 red and 6 black balls.

   (a) (7pts) If a ball is randomly selected from each urn, what is the probability that the two balls will be the same color?

   (b) (8pts) If 2 balls are drawn without replacement from each urn, what is the probability that 1 red and 3 black balls are selected?

   (c) (10pts) If a ball is drawn from $U_1$ and put into $U_2$, and then a second ball is picked from $U_2$; what is the probability that the second ball is red?

3. Three fair coins have their faces labeled with numbers 1 and 2 instead of heads and tails. Assume that you toss them and let $X$ be the sum of the three numbers on top. For instance, $X(111) = 3$ and $X(212) = 5$.

(a) (8pts) Find $\mathbb{P}(X > 4)$. Note that $X \in \{3, 4, 5, 6\}$

$$\mathbb{P}(X > 4) = \mathbb{P}(X = 5) + \mathbb{P}(X = 6)$$
$$= \mathbb{P}(\{122, 212, 221\}) + \mathbb{P}(\{222\}) = \underbrace{\frac{3}{2^3}}_{f(5)} + \underbrace{\frac{1}{2^3}}_{f(6)} = \frac{1}{2}$$

Since $\Omega = \{\omega_1\omega_2\omega_3 : \omega_i \in \{1,2\}, \forall i\}$
$\Rightarrow |\Omega| = 2^3$

(b) (9pts) Compute $\mathbb{E}(X)$.

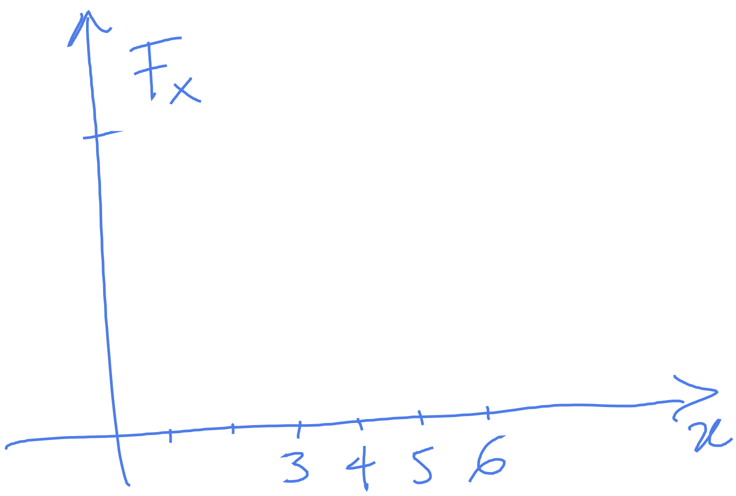
$\mathbb{P}(X = 3) = \mathbb{P}(\{111\}) = \frac{1}{2^3} = f(3)$ and $\mathbb{P}(X = 4) = \mathbb{P}(\{112, 121, 211\}) = \frac{3}{2^3} = f(4)$

Thus, $\mathbb{E}(X) = 3 \times \frac{1}{8} + 4 \times \frac{3}{8} + 5 \times \frac{3}{8} + 6 \times \frac{1}{8}$
$$= (3 + 12 + 15 + 6)/8 = 9/2.$$

A faster way is to observe that $X = 3 + \text{Bin}(3, 1/2)$.

(c) (8pts) Plot the distribution function of $X$.

$$F_X(x) = \begin{cases} 0, & x < 3 \\ 1/8, & 3 \le x < 4 \\ 1/2, & 4 \le x < 5 \\ 7/8, & 5 \le x < 6 \\ 1, & x \end{cases}$$



4. Determine whether the following statements are True or False. Justify your answer with a proof or a counterexample as appropriate.

   (a) (7pts) If $A$ and $B$ are independent events, then $A$ and $B^c$ are independent.

   (b) (6pts) If two random variables $X$ and $Y$ are constant, then they are independent.

   (c) (6pts) If $Y$ is a random variable with $\mathbb{P}(Y = 0) = 0$, then $\mathbb{E}(1/Y) = 1/\mathbb{E}(Y)$.

   (d) (6pts) If $X$ and $Y$ are $\text{Bin}(n, p)$ random variables, then $X + Y \sim \text{Bin}(n, p)$.