

# Midterm 1: Solutions to problems 1 and 2

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## Problem 1

Consider we regroup 40 numbers into two groups, one group contains the target 8 numbers and the other contains the rest 32 numbers.

1.

$$\mathbb{P}(\text{All 8 numbers selected}) = \frac{\binom{32}{0}\binom{8}{8}}{\binom{40}{8}}$$

2.

$$\mathbb{P}(\text{none of the 8 numbers is selected}) = \frac{\binom{32}{8}}{\binom{40}{8}}$$

3.

$$\begin{aligned} & \mathbb{P}(\text{At least 6 of the numbers is selected}) \\ = & \mathbb{P}(\text{6 of the numbers is selected}) \\ & + \mathbb{P}(\text{7 of the numbers is selected}) + \mathbb{P}(\text{8 of the numbers is selected}) \end{aligned}$$

$$= \frac{\binom{32}{2}\binom{8}{6} + \binom{32}{1}\binom{8}{7} + \binom{32}{0}\binom{8}{8}}{\binom{40}{8}}$$

## Problem 2

1.

$$\begin{aligned} & \mathbb{P}(\text{two balls has the same color}) \\ &= \mathbb{P}(\text{R1R2}) + \mathbb{P}(\text{B1B2}) \\ &= \frac{3}{6} \times \frac{4}{10} + \frac{3}{6} \times \frac{6}{10} = \frac{1}{2} \end{aligned}$$

2.

$$\begin{aligned} & \mathbb{P}(\text{1 red ball and 3 black balls}) \\ &= \mathbb{P}(\text{R1B1B2B2}) + \mathbb{P}(\text{R2B2B1B1}) \\ &= \frac{\binom{3}{1} \binom{3}{1} \binom{6}{2}}{\binom{6}{2} \binom{10}{2}} + \frac{\binom{4}{1} \binom{6}{1} \binom{3}{2}}{\binom{6}{2} \binom{10}{2}} \end{aligned}$$

3.

$$\begin{aligned} & \mathbb{P}(\text{second ball is red}) \\ &= \mathbb{P}(\text{second ball is red} \mid \text{ball from } U_1 \text{ is red})\mathbb{P}(\text{ball from } U_1 \text{ is red}) \\ & \quad + \mathbb{P}(\text{second ball is red} \mid \text{ball from } U_1 \text{ is black})\mathbb{P}(\text{ball from } U_1 \text{ is black}) \\ &= \frac{4}{11} \times \frac{3}{6} + \frac{5}{11} \times \frac{3}{6} \\ &= \frac{27}{66} = \frac{9}{22} \end{aligned}$$