Math 145, Winter 2020
March 17, 2020

TAKEHOME EXAM

Due: 03/18 at 03:00 p.m. Submission via Canvas

NAME(use CAPITAL letters, *first name first*):______________________________

ID# and Section (s001 or s002):____________________________

NAME(sign):________________________

Honor Statement: By signing above, I hereby declare that I solved this exam by my own, without any external collaboration (like friends, internet solutions, etc). If needed, I am allowed to use our main textbook and lecture notes only. I understand that the main purpose of this exam is to show how much I have learned in this course, holding myself to a high standard of academic integrity, and that suspected misconduct on this exam will be reported to the Office of Student Support and Judicial Affairs and, if established, will result in disciplinary sanctions up through Dismissal from the University and a grade penalty up to a grade of 'F' for the course.

Instructions: Read each question carefully. YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT. Clarity of your solutions may be a factor when determining credit. Unless directed to do so, do *not* prove any theorem or proposition seen in class, and do not evaluate complicated expressions to give the result as a fraction or a decimal number. However, if you are using any of the problems in the textbook, then you have to solve or prove it.

Write down your solutions and submit them on Canvas in pdf or jpg format.
Sign and submit this first page as well.

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4	
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6	
TOTAL	

Determine whether the following statements are True or False. Justify your answer with a proof or a counterexample as appropriate.

1. (15pts) If we choose 38 even positive integers, all less than 1000, then there will be two of them whose difference is at most 26.

2. (15pts) There are at least 102 distinct 4-tuples (a, b, c, d) of non-negative integers satisfying $a \leq 3,\ b \leq 4,\ c \leq 6,\ d \leq 7$ and $a + b + c + d = 16$.

3. (15pts) Given $n \geq 2$ and natural numbers $a_1, a_2, \ldots, a_n \geq 1$, the following statements are equivalent:

   (a) There exists a tree with n vertices $v_1, v_2, \ldots, v_n$ such that $\deg(v_k) = a_k$ for $k \leq n$.

   (b) $\sum_{k=1}^{n} a_k = 2n - 2$.

4. (20pts) Every maximal planar graph of order $n \geq 2020$ has $2n + 4$ regions.

   (Recall that a planar graph G is called *maximal planar* if the addition of any edge to G, without adding any new vertices, creates a nonplanar graph).

5. (20pts) If $1 \leq s \leq t$, then $\mathcal{M}(K_{s,t})$ has $t^{\underline{s}}$ vertices, $s(t-s)t^{\underline{s}}/2$ edges, and is regular.

   (Recall that the *matching graph* $\mathcal{M}(G)$ of a graph G has the maximum matchings of G as its vertices, and two vertices M_1 and M_2 of $\mathcal{M}(G)$ are adjacent if M_1 and M_2 differ in only one edge).

6. (15pts) $ES(n) \leq 1 + \binom{2r}{r}$, where $r = R_3(n-1, n) - 1$.