Lecture #2 (03/31/2021)

Recall that $S = \{\text{outcomes}\}$, $\mathcal{F} = \{\text{events}\}$, for finite S we are considering $\mathbb{P} : \mathcal{F} \to [0,1]$

$$E \mapsto \mathbb{P}(E) = |E| / |S|.$$

Ex 2.2: • $\mathbb{P}(2{:}2 \text{ boy-girl split in a 4-children family})$

$S = \Bigg\{$

BBBB	BBBG	BBGB	BGGG
BGBB	BGBG	BGGB	BGGG
GBBB	GBBG	GBGB	GBGG
GGBB	GGBB	GGGB	GGGG

$\Bigg\}$

$$|S| = 2 \times 2 \times 2 \times 2 = 2^4 = 16$$

$|E| = 6$, thus $\mathbb{P}(E) = |E| / |S| = 6/2^4 = 3/8$

• $\mathbb{P}(1{:}3 \text{ boy-girl split}) = 4/|S| = 4/16 = 1/4$.

• $\mathbb{P}(1{:}3 \text{ or } 3{:}1 \text{ boy-girl split}) = 8/16 = 1/2$.

Task: Work on Example 2.3. Identify S and E.

How to count?

Basic Principle of Counting: If an experiment consists of $K \geq 2$ stages and the 1^{st} stage has m_1 outcomes, while the 2^{nd} stage has m_2 outcomes regardless of the outcome at the 1^{st} stage, ..., and the K^{th} stage has m_K outcomes regardless of the previous ones, then the experiment as a whole has

$$m_1 \times m_2 \times \cdots \times m_K \text{ outcomes.}$$

Ex 2.4: Roll a die 4 times ($K=4$ stages).
What is the prob. to get different numers?
(for instance $2315,\ 6241 \in E$, but $3325 \notin E$)

Step 1: Find S. Each die has 6 outcomes

$$S = \{(a,b,c,d) : a,b,c,d \in \{1,2,3,4,5,6\}\}$$

Step 2: Calculate $|S|$. ($K=4$ stages) $m_1 = 6 = m_2$
also $m_3 = m_4 = 6$. So $|S| = m_1 \cdot m_2 \cdot m_3 \cdot m_4 = 6^4$.

Step 3: Find $|E|$. $E = \{\text{get diff. numbers}\}$

$K=4$. $\boxed{m_1}\boxed{m_2}\boxed{m_3}\boxed{m_4}$ $m_1 = 6,\ m_2 = 5,\ m_3 = 4,\ m_4 = 3$

So $|E| = m_1 m_2 m_3 m_4 = 6 \times 5 \times 4 \times 3$

thus $$P(E) = \frac{|E|}{|S|} = \frac{6\times5\times4\times3}{6^4} = \frac{5}{18} \approx 0.2778$$

DEFINITION: A permutation of n objects $a_1, a_2, \dots, a_n$ is a reordering of them.

Ex: How many permutations of $\{1,2,3\}$ ($n=3$) are there?

Ans: 123, 132, 213, 231, 312, 321

$\boxed{3}\boxed{2}\boxed{1}$ so there are $3\times2\times1 = 6$ permutations

In general: n objects

$\boxed{n}\boxed{n-1}\boxed{n-2}\boxed{n-3}\boxed{\cdots}\boxed{\cdots 2}\boxed{1}$ n slots

$\boxed{n}\boxed{n-1}\boxed{n-2}\boxed{\cdots}\boxed{n-(k-1)}$ K slots

PERMUTATIONS

Assume that we have n objects. The number of ways to fill n <u>ordered</u> slots with them is:

$$n\cdot(n-1)(n-2)\cdots 2\cdot 1 =: n! \qquad (n \text{ factorial})$$

while the number of ways to fill $k \le n$ slots is

$$n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

<u>Task</u>: Convince yourself.

<u>Ex 2.6</u>: Shuffle a deck of cards.

(a) $\mathbb{P}(\text{Top card is an Ace}) =$

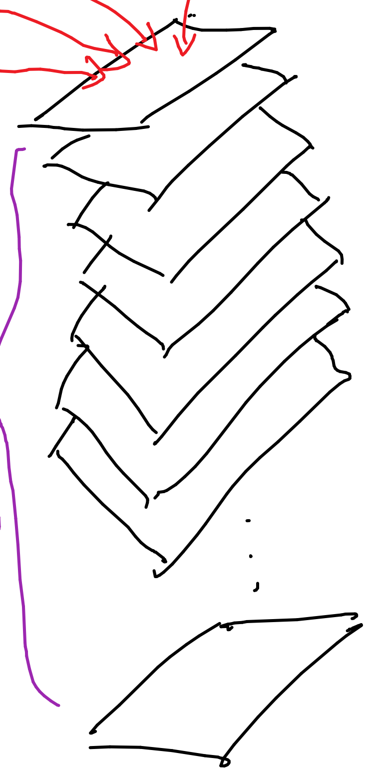
$S = \{\text{reordering of 52 cards}\} = \{\text{permutations of 52}\}$

So $|S| = 52! = 52\times 51\times 50\times 49\times\cdots\times 3\times 2\times 1$

$E = \{\text{Top card is either A}\heartsuit, \text{A}\spadesuit, \text{A}\diamondsuit \text{ or Aclubs}\}$

4 choices for 1st slot

51! choices for remaining slots.



51 cards remaining

Any perm. of 51 is fine

$$\mathbb{P}(E) = \frac{|E|}{|S|} = \frac{4\times 51!}{52!}$$

$$= \frac{4\times \cancel{51!}}{52\times\cancel{51!}} = \frac{4}{52} = \frac{1}{13}.$$

$n! = n * (n-1)!$ ← Property of !

task:

- $\mathbb{P}$(All cards of same suit end up next to each other) = ?
- $\mathbb{P}$(hearts are together) = ?