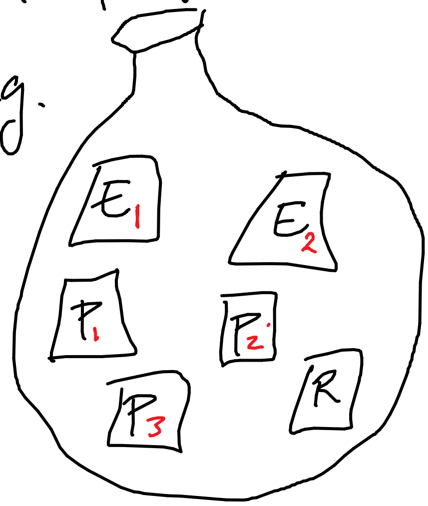
LECTURE #3 (04/02/2021)

Remarks: (a) When we say that a random choice is made, this means that all available choices are equally likely.

(b) Sampling with replacement refers to choosing at random, an object from a population, noting its properties, putting the object back into the population, and then repeating.

(c) Sampling without replacement omits the putting back part.

Ex 2.7: A bag has 6 pieces of paper:



Pull 6 pieces at random out of the bag.

(1) without

(2) with replacement

what is the prob. that these pieces in order, spell PEPPER ?

Sol (1) $|S| = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6!$

$S = \{$ permutations of 6 pieces of paper $\}$

Now, # good outcomes $= 3! \times 2! \times 1$

choice of P, choices of E, choices of R

$$\text{Prob} = \frac{3! \times 2!}{6!} = \frac{1}{60}$$

(2) $|S| = 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6$,

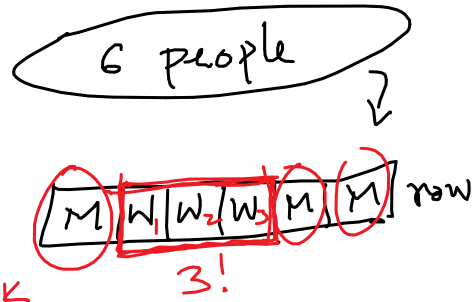
R R R R R R

$$\text{Prob} = \frac{\#\text{good}}{6^6} = \frac{3 \times 2 \times 3 \times 3 \times 2 \times 1}{6^6} = \frac{1}{2 \cdot 6^3}$$

Ex 2.8: Sit 3 men and 3 women at random

(1) In a row of chairs (2) around a table

$\mathbb{P}$(all women sit together)



Sol (1) $S = \{$permutations of 6 people$\}$

$|S| = 6!$

$\#$good outcomes $= \underset{\text{perm. of W}}{3!} \times \underset{\text{perm of 3M and 1 block}}{4!}$

$$\text{prob} = \frac{3! \times 4!}{6!} = \frac{1}{5}.$$

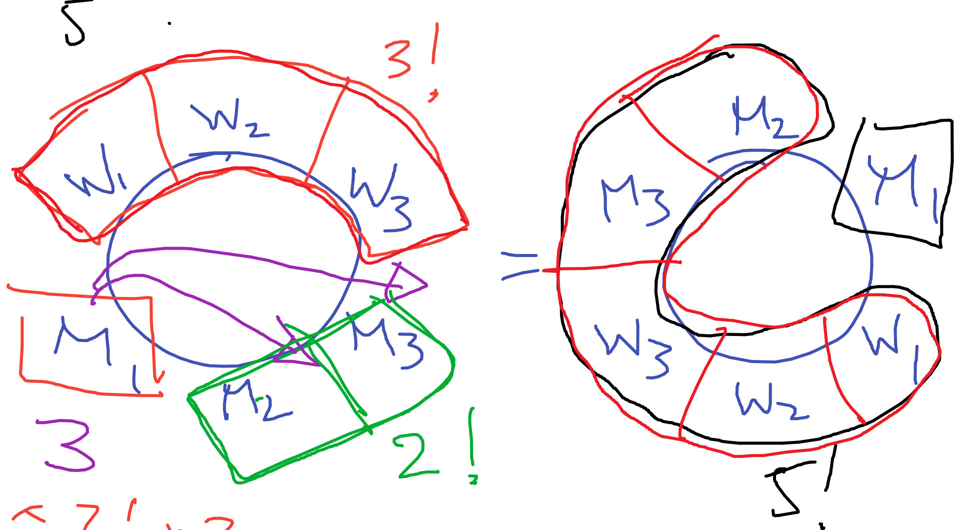
WWW MMM

MM WWW M

MMM WWW

(2) Table

Fix a man M_1



$\#$ good outcomes $= 3! \times 2! \times 3$

Also $S = \{$permutations of 5 remaining people$\}$

So $|S| = 5!$

and $$\text{prob} = \frac{3! \times 2! \times 3}{5!} = \frac{3}{10}.$$

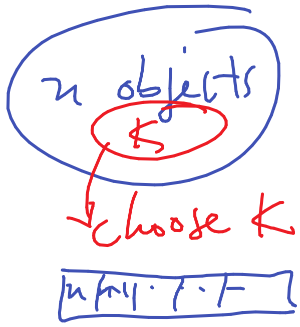
Task: Work Example 2.9.

COMBINATIONS

As permutations $123 \neq 132$ (order matters), but as a combination $\{1,2,3\} = \{1,3,2\}$ (order doesn't matter)

Let $\binom{n}{k}$ be the number of different subsets with k elements o a set with n elements. Then

$$\binom{n}{k} = \frac{n(n-1)\cdots(n-k+1)}{k!} = \frac{n!}{k!(n-k)!}$$



We call $\binom{n}{k}$ = n choose k or a binomial coefficient.

THM (Binomial theorem): If $x, y \in \mathbb{R}$, $n \in \mathbb{N}$ then

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

Also

$$\binom{n}{0} = \frac{n!}{0!\,n!} = 1$$

$$\binom{n}{n} = \frac{n!}{n!\,0!} = 1.$$

$\boxed{\text{since } 0! = 1}$

And we have the symmetry $\binom{n}{k} = \binom{n}{n-k}$