

Math 135A, Spring 2021
March 30, 2021

HOMEWORK 1

The homeworks will NOT be collected. You are strongly encouraged to do the assigned problems (taken from the 10th Ed. of Ross' book) on your own by the due date. Some solutions will be provided, but will not be carefully proofread, so check for mistakes!

Problems: 1,2, 3, 4, 7, 8, 9, 10, 18, 19, 20 and 21

These types of problems are called *combinatorial problems*.

Problems

1.
 - a. How many different 7-place license plates are possible if the first 2 places are for letters and the other 5 for numbers?
 - b. Repeat part (a) under the assumption that no letter or number can be repeated in a single license plate.
2. How many outcome sequences are possible when a die is rolled four times, where we say, for instance, that the outcome is 3, 4, 3, 1 if the first roll landed on 3, the second on 4, the third on 3, and the fourth on 1?
3. Twenty workers are to be assigned to 20 different jobs, one to each job. How many different assignments are possible?
4. John, Jim, Jay, and Jack have formed a band consisting of 4 instruments. If each of the boys can play all 4 instruments, how many different arrangements are possible? What if John and Jim can play all 4 instruments, but Jay and Jack can each play only piano and drums?

How many kittens did the traveler meet?

7.
 - a. In how many ways can 3 boys and 3 girls sit in a row?
 - b. In how many ways can 3 boys and 3 girls sit in a row if the boys and the girls are each to sit together?
 - c. In how many ways if only the boys must sit together?
 - d. In how many ways if no two people of the same sex are allowed to sit together?

8. When all letters are used, how many different letter arrangements can be made from the letters
 - a. Fluke?
 - b. Propose?
 - c. Mississippi?
 - d. Arrange?

9. A child has 12 blocks, of which 6 are black, 4 are red, 1 is white, and 1 is blue. If the child puts the blocks in a line, how many arrangements are possible?
10. In how many ways can 8 people be seated in a row if
 - a. there are no restrictions on the seating arrangement?
 - b. persons *A* and *B* must sit next to each other?
 - c. there are 4 men and 4 women and no 2 men or 2 women can sit next to each other?
 - d. there are 5 men and they must sit next to one another?

How many results are possible?

18. A student has to sell 2 books from a collection of 6 math, 7 science, and 4 economics books. How many choices are possible if
 - a. both books are to be on the same subject?
 - b. the books are to be on different subjects?

19. Seven different gifts are to be distributed among 10 children. How many distinct results are possible if no child is to receive more than one gift?
20. A committee of 7, consisting of 2 Republicans, 2 Democrats, and 3 Independents, is to be chosen from a group of 5 Republicans, 6 Democrats, and 4 Independents. How many committees are possible?
21. From a group of 8 women and 6 men, a committee consisting of 3 men and 3 women is to be formed. How many different committees are possible if
 - a. 2 of the men refuse to serve together?
 - b. 2 of the women refuse to serve together?
 - c. 1 man and 1 woman refuse to serve together?

22. A person has 8 friends, of whom 5 will be invited to a party.
 - a. How many choices are there if 2 of the friends are feuding and

Other Section of Problems: 1, 2, 3, 4, 5.

1. A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment that consists of taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat when the second marble is drawn without replacing the first marble.
2. In an experiment, die is rolled continually until a 6 appears, at which point the experiment stops. What is the sample space of this experiment? Let E_n denote the event that n rolls are necessary to complete the experiment. What points of the sample space are contained in E_n ? What is $\left(\bigcup_1^{\infty} E_n\right)^c$?

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3. Two dice are thrown. Let E be the event that the sum of the dice is odd, let F be the event that at least one of the dice lands on 1, and let G be the event that the sum is 5. Describe the events $EF, E \cup F, FG, EF^c$, and EFG .
4. A, B , and C take turns flipping a coin. The first one to get a head wins. The sample space of this experiment can be defined by

$$S = \begin{cases} 1,01,001,0001,\dots \\ 0000\dots \end{cases}$$

- a. Interpret the sample space.
- b. Define the following events in terms of S : 10pt
 - i. A wins = A .
 - ii. B wins = B .
 - iii. $(A \cup B)^c$.

Assume that A flips first, then B , then C , then A , and so on.

5. A system is composed of 5 components, each of which is either working or failed. Consider an experiment that consists of observing the status of each component, and let the outcome of the experiment be given by the vector $(x_1, x_2, x_3, x_4, x_5)$, where x_i is equal to 1 if component i is working and is equal to 0 if component i is failed.
 - a. How many outcomes are in the sample space of this experiment?
 - b. Suppose that the system will work if components 1 and 2 are both working, or if components 3 and 4 are both working, or if components 1, 3, and 5 are all working. Let W be the event that the system will work. Specify all the outcomes in W .
 - c. Let A be the event that components 4 and 5 are both failed. How many outcomes are contained in the event A ?