Mat 21B (Integral Calculus)

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INTRODUCTION

Recall from 21A that given a function $f: I \to \mathbb{R}$ (I is an open interval), the derivative of f at $x \in I$ is

$$f'(x) = \frac{df}{dx}(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h},$$

if the limit exists.

We learned some properties:

ⓐ $(f \pm g)' = f' \pm g'$, ⓑ $(fg)' = f'g + fg'$

ⓒ $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$, ⓓ $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

Ex: $\frac{d}{dx}(x^n) = n x^{n-1}$, $\frac{d}{dx}(\sin x) = \cos x$,

$\frac{d}{dx}(e^x) = e^x$, $\frac{d}{dx}(n^x) = \ln(n) \cdot n^x$. n is a positive constant.

So, given f we were interested in finding f'.

In this course, we are going somehow in the other direction.

§ 4.8 ANTIDERIVATIVES

DEF: A function F is an antiderivative of $f: I \to \mathbb{R}$ if $F'(x) = f(x)$, $\forall x \in I$.

The process of finding F is called antidifferentiation.

Ex: Find antiderivatives for

ⓐ $f(x) = 2x$ ⓑ $\frac{1}{x} + 2e^{2x} = g(x)$

Sol: ⓐ Since $(x^2)' = 2x$, then $F(x) = x^2$ is an antiderivative of f.

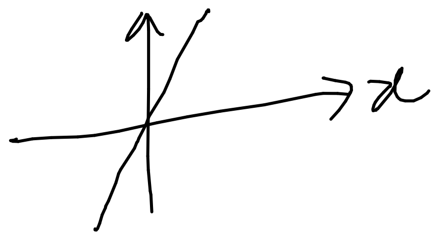
ⓑ $G(x) = \ln|x| + e^{2x}$, $[\mathrm{Dom}(g) = \mathbb{R} \setminus \{0\}]$

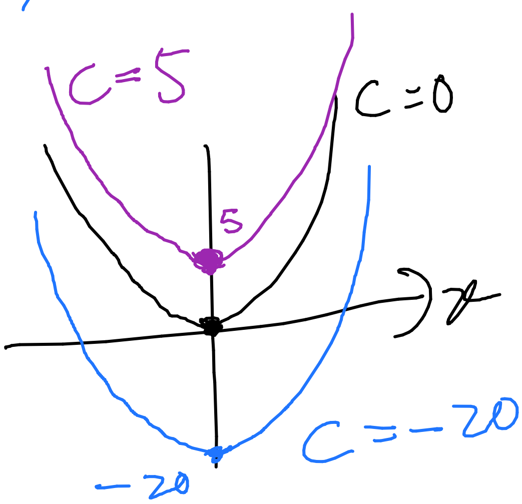
since $G'(x) = g(x)$.

Q: Are antiderivatives unique? ANS: No!

THM If F is an antiderivative of f, then $F(x) + C$ is an antiderivative of f, for any constant C, and those are the only ones.

Ex ⓐ $f(x) = 2x$ $F(x) = x^2 + C$





Read Table 4.2.