

Math 21B-B, Fall 2020
December 17, 2020

TAKEHOME FINAL
Due: Today 12/17 at 20:00

NAME(use CAPITAL letters, *first name first*): _____

NAME(sign): _____

ID#: _____

HONOR STATEMENT: By signing this paper, I hereby declare that I solved this exam by my own, without any external collaboration (like friends, internet solutions, etc). If needed, I am allowed to use our lecture notes only. I understand that the main purpose of this exam is to show how much I have learned in this course, holding myself to a high standard of academic integrity, and that suspected misconduct on this exam will be reported to the Office of Student Support and Judicial Affairs and, if established, will result in disciplinary sanctions up through Dismissal from the University and a grade penalty up to a grade of 'F' for the course.

Instructions: Read each question carefully and answer it in the space provided, or answer it in a separate paper and sign that paper; it is optional to print this exam. **YOU MUST SHOW ALL YOUR WORK TO RECEIVE FULL CREDIT.** Clarity of your solutions may be a factor when determining credit. Unless directed to do so, do *not* evaluate complicated expressions to give the result as a fraction or a decimal number. If you are using any of the problems in the textbook, then you have to solve it.

To deliver: Submit your solutions on Gradescope, as you did with the MT1. Sign and submit the honor statement (write it down by hand, if needed).

Make sure that you have a total of 7 pages (including this one) with 6 problems.

1	
2	
3	
4	
5	
6	
TOTAL	200 pts

1. (a) *(16pts)* For what values of m do the line $y = mx$ and the curve $y = \frac{x}{x^2 + 1}$ enclose a region? Find the area of the region.

- (b) *(17pts)* Find the volume obtained by rotating about the y -axis, the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$.

2. (a) *(16pts)* A chain lying on the ground is 10 m long and its mass is 80 kg. How much work is required to raise one end of the chain to a height of 6 m?

- (b) *(17pts)* Evaluate the integral $\int_{-\pi}^{\pi} x \sin x \cos x dx$. Then, write down the error $|E_S|$ obtained by approximating it using Simpson's Rule with $n = 8$.
(You do not need to simplify the estimation, and recall that $\cos \frac{\pi}{4} = \sin \frac{3\pi}{4} = \frac{\sqrt{2}}{2}$).

3. For each of following integrals either evaluate it or show that it diverges.

(a) $\langle 16pts \rangle \int_0^2 \frac{x+1}{\sqrt{4-x^2}} dx$

(b) $\langle 17pts \rangle \int_0^\infty \frac{dx}{x^2+2x-3}$

4. (a) $\langle 16pts \rangle$ Find the arc length of the curve

$$y = \frac{1}{6}(x^2 + 4)^{3/2}, \quad x \in [0, 3].$$

- (b) $\langle 17pts \rangle$ Find d^2y/dx^2 and sketch the curve

$$x = 1 + \frac{1}{t^2}, \quad y = 1 - \frac{3}{t}, \quad t \in [1, 9].$$

5. (a) $\langle 17pts \rangle$ Find the area of the surface generated by revolving the curve

$$x = 4\sqrt{t}, \quad y = \frac{t^3}{3} + \frac{1}{2t^2}, \quad t \in [1, 4]$$

about the x -axis.

- (b) $\langle 17pts \rangle$ Find a Cartesian equation and calculate the arc length of the curve given by the polar equation $r = 2 \sin \theta + 2 \cos \theta$, $0 \leq \theta \leq \pi/2$.

6. Determine whether the following statements are True or False. Justify your answers.

(a) *(17pts)* If $\int_a^\infty f(x) dx$ and $\int_a^\infty g(x) dx$ are both divergent, then $\int_a^\infty [f(x) + g(x)] dx$ is divergent (a is a fixed constant).

(b) *(17pts)* If $a_n = \sum_{i=1}^n \ln \sqrt[n]{1 + \frac{i}{n}}$ for $n \in \mathbb{N}$, then $\lim_{n \rightarrow \infty} a_n = \infty$.