Math 21B, Fall 2020
November 4, 2020

HOMEWORK 2

Due: 11/11/2020 at 17:00

This homework has 2 pages, but you have to solve 15 problems in total. After the due, only 3 problems will be randomly selected and graded.

The instructions are:

- Students in Section B: Solve all problems from PART I, but NOT the additional problems in PART II.
- Students in Section 0U1: Solve all problems (including the ones in PART II), except problems 1, 2 and 3 from PART I.

PART I: FROM SCANNED PROBLEMS (at our website)

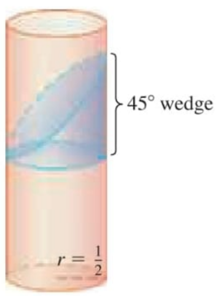
1. 7.1.18
2. 7.1.60
3. 7.2.22
4. 6.1.48
5. 6.1.55
6. 6.3.14
7. 6.3.24
8. 6.4.26
9. 6.4.31
10. 6.5.5
11. 6.5.24
12. 6.5.42

The remaining three problems for Part I are the ones in red circle in the next picture, namely, 5, 6 and 8. **Caution**: Do NOT use Pappus's Theorem, since we have not covered it. Use your creativity instead.

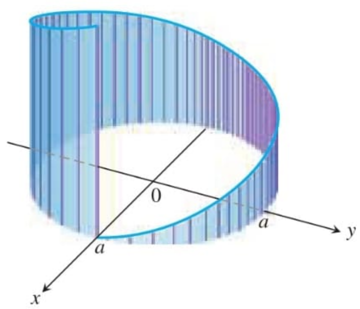
b. Generalize the result in part (a).

5. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x$ and $y = x^2$ about the line $y = x$.

6. Consider a right-circular cylinder of diameter 1. Form a wedge by making one slice parallel to the base of the cylinder completely through the cylinder, and another slice at an angle of 45° to the first slice and intersecting the first slice at the opposite edge of the cylinder (see accompanying diagram). Find the volume of the wedge.



8. At points on a circle of radius a, line segments are drawn perpendicular to the plane of the circle, the perpendicular at each point P being of length ks, where s is the length of the arc of the circle measured counterclockwise from $(a, 0)$ to P and k is a positive constant, as shown here. Find the area of the surface formed by the perpendiculars along the arc beginning at $(a, 0)$ and extending once around the circle.



PART II: ADDITIONAL PROBLEMS

Solve the three problems in red circle below, namely, 9, 7 and 18.

Work

9. A particle of mass m starts from rest at time $t = 0$ and is moved along the x-axis with constant acceleration a from $x = 0$ to $x = h$ against a variable force of magnitude $F(t) = t^2$. Find the work done.

10. Work and kinetic energy Suppose a 1.6-oz golf ball is placed on a vertical spring with force constant $k = 2$ lb/in. The spring is compressed 6 in. and released. About how high does the ball go (measured from the spring's rest position)?

Centers of Mass

11. Find the centroid of the region bounded below by the x-axis and above by the curve $y = 1 - x^n$, n an even positive integer. What is the limiting position of the centroid as $n \to \infty$?

12. If you haul a telephone pole on a two-wheeled carriage behind

CHAPTER 7 Additional and Advanced Exercises

Limits

Find the limits in Exercises 1–6.

1. $\lim_{b \to 1^-} \int_0^b \frac{dx}{\sqrt{1 - x^2}}$

2. $\lim_{x \to \infty} \frac{1}{x} \int_0^x \tan^{-1} t \, dt$

3. $\lim_{x \to 0^+} (\cos \sqrt{x})^{1/x}$

4. $\lim_{x \to \infty} (x + e^x)^{2/x}$

5. $\lim_{n \to \infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n} \right)$

6. $\lim_{n \to \infty} \frac{1}{n} \left(e^{1/n} + e^{2/n} + \cdots + e^{(n-1)/n} + e^{n/n} \right)$

7. Let $A(t)$ be the area of the region in the first quadrant enclosed by the coordinate axes, the curve $y = e^{-x}$, and the vertical line $x = t$, $t > 0$. Let $V(t)$ be the volume of the solid generated by revolving the region about the x-axis. Find the following limits.

a. $\lim_{t \to \infty} A(t)$ **b.** $\lim_{t \to \infty} V(t)/A(t)$ **c.** $\lim_{t \to 0^+} V(t)/A(t)$

8. Varying a logarithm's base

a. Find $\lim \log_a 2$ as $a \to 0^+, 1^-, 1^+$, and ∞.

T **b.** Graph $y = \log_a 2$ as a function of a over the interval

18. Let g be a function that is differentiable throughout an open interval containing the origin. Suppose g has the following properties:

i) $g(x + y) = \frac{g(x) + g(y)}{1 - g(x)g(y)}$ for all real numbers x, y, and $x + y$ in the domain of g.

ii) $\lim_{h \to 0} g(h) = 0$

iii) $\lim_{h \to 0} \frac{g(h)}{h} = 1$

a. Show that $g(0) = 0$.

b. Show that $g'(x) = 1 + [g(x)]^2$.

c. Find $g(x)$ by solving the differential equation in part (b).

19. Center of mass Find the center of mass of a thin plate of constant density covering the region in the first and fourth quadrants enclosed by the curves $y = 1/(1 + x^2)$ and $y = -1/(1 + x^2)$ and by the lines $x = 0$ and $x = 1$.

20. Solid of revolution The region between the curve $y = 1/(2\sqrt{x})$ and the x-axis from $x = 1/4$ to $x = 4$ is revolved about the x-axis to generate a solid.