

4. Determine whether the following statements are True or False and give reasons for your answers.

(a) There are at least two different curves $y = y(x)$ whose arc length for $1 \leq x \leq 9$ is

$$L = \int_1^9 \sqrt{1 + \frac{1}{4x}} dx$$

True: Indeed, there are infinitely many, for instance,
 $y_1 = \sqrt{x^2} + \pi$ and $y_2 = -\sqrt{x^2}$

Both satisfy $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$, so $\left(\frac{dy}{dx}\right)^2 = \frac{1}{4x}$

and they have the same $L = \int_1^9 \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$.

(b)

$$\int_{-\pi}^{\pi} \sin^{2020}(x) dx = -\frac{2\pi}{2021}.$$

(c) Every integer $n \geq 2$ satisfies the inequality $\ln n < \sum_{k=1}^{n-1} \frac{1}{k}$.